Exercise 41

Find the numbers at which f is discontinuous. At which of these numbers is f continuous from the right, from the left, or neither? Sketch the graph of f.

$$f(x) = \begin{cases} x^2 & \text{if } x < -1 \\ x & \text{if } -1 \leq x < 1 \\ 1/x & \text{if } x \geq 1 \end{cases}$$

Solution

x^2 and x are polynomials and are continuous at all numbers by Theorem 7. $1/x$ is a rational function and is continuous for all $x \geq 1$ by Theorem 7. Any points of discontinuity, then, can only occur at the endpoints of the intervals that these functions are defined on. Check $x = -1$ first.

$$\lim_{x \to -1^-} f(x) \stackrel{?}{=} \lim_{x \to -1^+} f(x) \stackrel{?}{=} f(-1)$$

$$\lim_{x \to -1^-} x^2 \stackrel{?}{=} \lim_{x \to -1^+} x \stackrel{?}{=} x\bigg|_{x=-1}$$

$$(-1)^2 \stackrel{?}{=} (-1) \stackrel{?}{=} -1$$

$$1 \neq -1 = -1$$

The condition for $f(x)$ to be continuous at $x = -1$ is not satisfied. Therefore, $f(x)$ is discontinuous at $x = -1$ but continuous from the right. Check $x = 1$ next.

$$\lim_{x \to 1^-} f(x) \stackrel{?}{=} \lim_{x \to 1^+} f(x) \stackrel{?}{=} f(1)$$

$$\lim_{x \to 1^-} x \stackrel{?}{=} \lim_{x \to 1^+} \frac{1}{x} \stackrel{?}{=} \frac{1}{x}\bigg|_{x=1}$$

$$1 = 1 = 1$$

The condition for $f(x)$ to be continuous at $x = 1$ is satisfied, so there's no point of discontinuity at $x = 1$.

Below is a graph of $f(x)$ versus x.